

Model-based Dynamic Gait Generation for a Leg-wheel Transformable Robot

Hung-Sheng Lin, Wei-Hsi Chen, and Pei-Chun Lin

Abstract—We report on the model-based approach to dynamic trotting and pronking gait generation for a leg-wheel transformable robot. The rolling spring-loaded inverted pendulum (R-SLIP) model served as the template for robot locomotion by programming the robot’s motion according to the stable fixed-point trajectories of the model. Two strategies are developed to match the robot leg motions to the virtual model leg: First, using the two active degrees of freedom on each leg to simulate the passive spring effect of the R-SLIP model leg. Second, installing a torsion spring on the leg-wheel module to render the leg-wheel morphology identical to that of the model leg. This model-based approach to dynamic behavior generation for the robot is experimentally evaluated. The robot can successfully generate an R-SLIP-like stable pronking gait with a flight phase.

I. INTRODUCTION

The environment around humans is full of unknowns as well as various obstacles. How to build a robot that can operate on various kinds of terrain is therefore a challenging task. On flat terrains, which are mostly artificial, wheeled robots are preferable because their movement is fast, smooth, and power-efficient. In contrast, on rugged terrains, which are mostly natural, the robot’s ability to negotiate obstacles is crucial. In this case, animals are ideal models for inspiration. As part of the evolutionary process, ground animals mostly developed a legged morphology. Through the careful integration of design and coordination control, such animals exhibit robust, agile, dynamic, and incomparable locomotive performance on a wide range of terrains. As a result, robots with a legged morphology are constructed and evaluated in terms of ground locomotion, mainly with regard to two crucial issues: rough terrain negotiability and dynamic maneuvering.

Legged robots with different numbers of legs have different motion characteristics in relation to the above two issues. In general, it is easier to develop rough terrain negotiability and dynamical maneuvering for hexapod robots owing to their intrinsically stable locomotion with multiple ground-contact legs. The Sprawl series [1] and the RHex [2, 3] use a well-designed mechanical structure and tripod locomotion strategy to achieve high-speed running. In addition, by taking advantage of compliant legs and individually controlled leg motion, the RHex can also perform other kinds of dynamic maneuvering, such as self-righting [4] and leaping [5, 6]. In



Fig. 1. Photo of the *TurboQuad* robot.

contrast, it is more challenging to develop rough terrain negotiability and dynamic maneuvering for quadruped robots owing to the lower number of legs available for “stable” maneuvering. The Stanford LittleDog uses both static and dynamic gaits to negotiate challenging terrains with a speed of up to 0.237 m/s [7]. The Scout II can perform a bounding gait with a speed of up to 1.3 m/s on flat ground [8]. Its successor, the leg-wheel hybrid robot PAW, can also perform a bounding gait with a clear flight phase [9]. The quadruped robot Tekken can walk on irregular terrains in an outdoor environment with a speed of up to 0.95 m/s [10]. The Cheetah-cub, a compliant quadruped robot, can perform trotting with short flight phases and with speeds of up to 1.42 m/s [11]. The KOLT quadruped robot uses an analytical model of the electro-pneumatic leg thrusting system and can show a dynamic trotting gait with a speed of up to 1.1 m/s [12]. The HyQ uses compliant control based on a virtual spring abstraction to perform a trotting gait [13]. The MIT Cheetah can achieve high-speed trot-running of up to 6 m/s with a hierarchical controller with programmable virtual leg compliance, tunable stance trajectory design and a gait-pattern modulation [14]. The Cheetah and WildCat developed by Boston Dynamics can also perform dynamic running, but only very limited technical information has so far been released.

Recently, we developed a leg-wheel transformable robot, *TurboQuad*, as shown in Fig.1, and its specification is

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TABLE I ROBOT SPECIFICATIONS

LENGTH AND WEIGHT		
Length	Body	0.71 m
	Hip-to-hip	0.454 m
Width	Body	0.194 m
	Leg-to-leg	0.371 m
Height	Body	0.16 m
	Ground to hip (legged mode)	0.14 m
	Ground clearance (legged mode)	0.1 m
Leg-wheel diameter	0.21 m	
Weight	18.5 kg	
ACTUATOR AND SENSORS		
	Position	Number
60W DC motor	2 DOF mechanism	8
Encoder	Motor	8
Hall-effect	2 DOF mechanism	4
Temperature sensor	Computing unit	1
Battery current measurement	Power output	1
Battery voltage measurement	Power output	1
6-axis IMU	COM	1
2-axis inclinometer	Front side	1

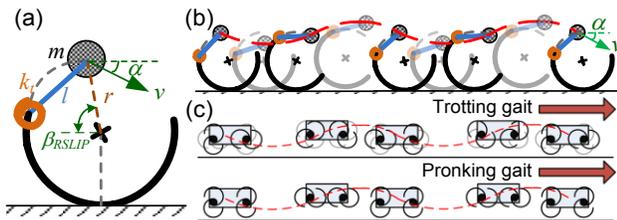


Fig. 2. The R-SLIP model: (a) Parameters and initial conditions of the model; (b) its dynamic motion; and (c) the corresponding robot trotting and pronking motions.

detailed in Table I. The robot is equipped with a special transformation mechanism, which takes advantage of the half-circular leg to negotiate obstacles and of the wheels for power efficiency and motion smoothness [15]. Compared to its processor, *Quattroped* [16], the *TurboQuad* has a new leg-wheel transformation mechanism and control strategy. These two innovations serve as the infrastructure for developing fast gait transition, dynamic leg coordination, and leg-wheel transformation on the robot. Here, aiming at extending the demonstrated functionality of both the wheeled mode on flat terrain and the quasi-static legged mode on rough terrain of the *Quattroped* [15], we take advantage of the new infrastructure of the *TurboQuad* and report on our model-based approach to exciting dynamic trotting and pronking gaits on the robot. The R-SLIP model [17] shown in Fig. 2(a) served as the “template” for the robot, the “anchor” for motion generation [18]. Recently, we used a similar model-based strategy to generate tripod running with the RHex-style robot with various designable speeds without tuning and gait optimization efforts [17]. In that case, the mapping of the

virtual model leg to the empirical robot leg was achieved by specifically designing a model leg that can replicate the passive dynamic properties of the robot’s compliant legs. Here, because the R-SLIP model can replicate the behavior of rolling quite well, it served as the “template” in the following development. In the meantime, because the leg-wheel modules on the *TurboQuad* have to be stiff so that the wheel does not deform, a new strategy is required to simulate the compliant effect of the R-SLIP model. In this work, we investigate two different approaches with which to achieve this goal: one is to design the leg motion as a passive spring, while the other is to install a torsion spring in the leg-wheel module.

The paper is organized as follows. Section II describes the strategy of using the R-SLIP model as a template. Section III introduces two methods with which to achieve motion mapping between the model leg and the robot leg. Section IV reports the results of the experimental evaluation, while Section V concludes the work.

II. USING THE R-SLIP MODEL AS THE TEMPLATE

The dynamic behaviors of the robot can be excited via several different methods. Parameter tuning or optimization is one of the more widely used approaches. This process usually involves two steps: The first step is to define and parameterize the leg motion pattern, and the second step is to search for the right combination of the defined parameters of the robot. The advantage of this approach is that there is no need to understand the underlying dynamics of the robot itself. The tuning and optimization process would maneuver the parameters to the desired behaviors if the defined motion pattern includes dynamic behaviors. The drawback to this approach is that very limited insight into the robot’s dynamics can be revealed during the process, and the adequate behavior may not be easily initiated if the defined motion pattern is not set within the right parameter space.

In this paper, the model-based approach is utilized. As mentioned in the introduction, we used the R-SLIP model as the similar model-based strategy to generate tripod running on the RHex-style robot with various designable speeds without tuning and gait optimization efforts [17]. In that work, the three legs of the tripod are driven to simulate the effect of the virtual leg of the model. Owing to very small pitch and roll variations, the two-dimensional sagittal-plane model R-SLIP is sufficient for investigating dynamic locomotion in the forward direction, mainly running. One interesting phenomenon of insects is that they use tripod locomotion as their main gait for locomotion, whether it be walking or running. In contrast, quadruped animals have various different gaits, including walking, trotting, pronking, bounding, etc. Here, because walking is generally quasi-static and bounding requires a model with rigid body effect, the trotting and pronking gaits are investigated. Since the body pitch and roll of the animals during these two gaits only have small variations, the “template” for the robot doesn’t need a rigid body. In addition, although the animal uses a different

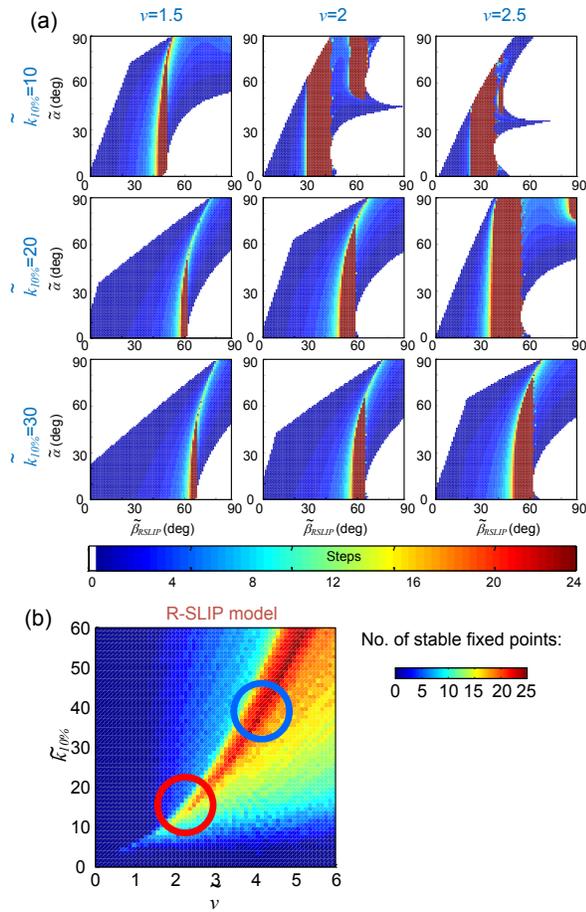


Fig. 3. Stability analysis of the R-SLIP model: (a) Steps-to-fall analysis of the model; and (b) number of fixed points versus speed and stiffness conditions.

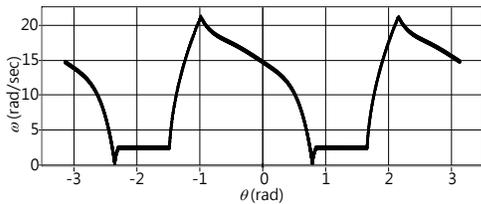


Fig. 4. The plot of leg angular speed (ω) versus leg orientation (θ).

number of legs to contact the ground when using these two gaits, the motion profile of the animal in the sagittal plane is similar. Thus, it is feasible to use one reduced-order model to act as the “template” for the robot. Our past experience reveals that the R-SLIP model is quite adequate for simulating the dynamic running of the robot with rolling contact. Therefore, instead of using an ordinary SLIP model [19], the R-SLIP model is utilized. As shown in Fig. 2(a), the R-SLIP model has four intrinsic parameters: radius of the circular rim (r), stiffness of the torsional spring (k_t), mass (m), and distance between the torsion spring and the mass (l). The model has three initial conditions given at the moment of touchdown (i.e., beginning of the stance phase), which include the landing angle (β), touchdown speed (v), and touchdown angle formed by the touchdown velocity and

horizontal line (α). Similar to the SLIP model, the R-SLIP model has a self-stable dynamic running gait, which includes both a stance phase and a flight phase as shown in Fig. 2(b). The motion sequence of the robot performing R-SLIP-like trotting and pronking is depicted in Fig. 2(c). The advantage of pronking is that it uses all four legs simultaneously, so the robot in pronking has larger actuation power density than the robot in trotting, which uses two legs at each contact and alternates periodically. On the other hand, the drawback of the pronking gait is that it requires a faster leg reposition motion for the next touchdown.

The model-based strategy for exciting dynamic trotting and pronking behaviors in the robot is to match the robot’s leg trajectories to the stable trajectories of the R-SLIP model. In [17], we have reported the dimensionless analysis of the R-SLIP model, which reveals the stable and unstable fixed points of the model with a wide range of parameters and initial conditions as shown in Fig. 3. Thus, by using this figure for design guidance, the process followed includes two steps: First, identify the suitable stable trajectory of the R-SLIP model as guidance. Second, match the leg motion to that of the R-SLIP leg. Figure 3 reveals that for gaits with a higher running speed, the larger the stiffness of torsion spring is needed. In the robot’s case, the robot in pronking has larger leg stiffness than that in trotting, so the forward speed of the robot with a pronking gait should be set higher than that with a trotting gait. Therefore, the suitable dimensionless stiffness of the torsion spring for pronking (ex. the blue circle area with more fixed points) should be set higher than for a trotting gait (ex. the red circle area). As a result, the suitable operation points for a pronking gait and for trotting must be different. Empirically, the stiffness requirements for trotting and pronking may not be satisfied by a given stiffness value.

Various robot’s empirical characteristics limit the choices of feasible operation points. The mass of the model, equal to the mass of the robot, should be treated a priori. This rule is also applied to the legs’ geometrical dimensions. The feasible speed of the robot is also constrained by the motor power and natural dynamics. The former limited the top speed of the robot, while the latter requires the robot’s speed to pass a certain minimum speed. Based on experience with the robot operated in wheeled mode and legged mode with a walking gait, the speed of the robot in trotting is set at around 2 m/s. By using the factors listed in Table I, the suitable stiffness of the torsion spring for trotting gait is set as 23.24 N · m/rad. Following this, the reset of unknown initial conditions such as touchdown angle (α) and landing angle (β) can be decided.

The R-SLIP-based leg profile of the robot should comply with the existing control architecture. The *TurboQuad* has a Central Pattern Generator (CPG), which is composed of multiple oscillators and serves as the multi-leg coordination mechanism. The implementation of the R-SLIP trajectory in a specific trotting or pronking gait is feasible but, instead of using time as the independent variable, the leg orientation (θ) is adopted. The remaining variables such as leg speed, leg length, and leg length rate are represented as functions of the

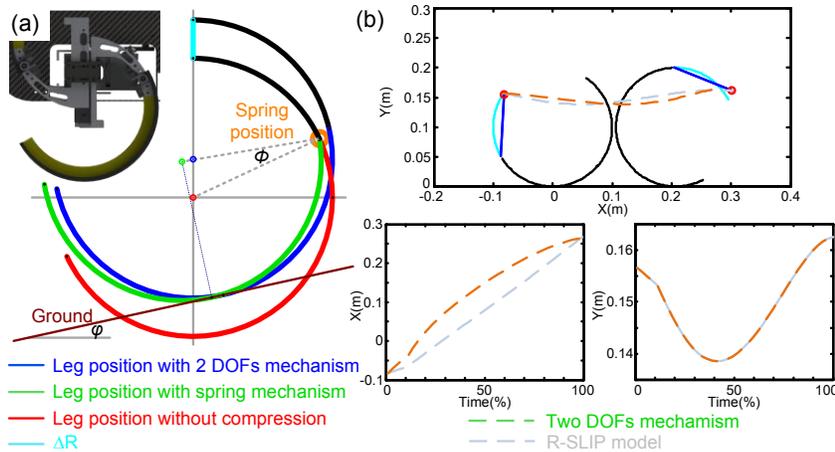


Fig. 5. Leg motion generation: (a) Geometrical configurations of the R-SLIP model and the robot leg; and (b) trajectory comparison between the model and the robot.

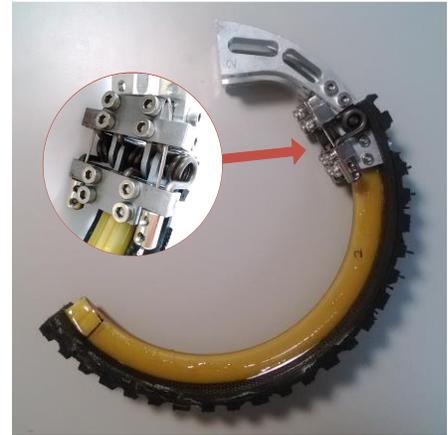


Fig. 6. The robot leg with a torsion spring.

leg orientation. Below is an example of the leg speed, and its state versus the leg orientation is plotted in Fig. 4.

$$\omega = f(\theta) \quad (1)$$

By building the database of $\omega = f(\theta)$ under different initial conditions, the robot can be set to run with different conditions according to the corresponding R-SLIP model in real-time as well.

III. THE R-SLIP-BASED LEG MOTION GENERATION: TWO APPROACHES

There are two methods with which to match the robot leg behavior to that of the R-SLIP leg. One is to use the robot's 2-DOF leg-wheel mechanism. By carefully matching the rotational translational DOFs in the sagittal plane, the "spring-like" leg motion of the R-SLIP can be simulated on the robot. The other method is to install a passive torsion spring in the leg-wheel module. In this case, the leg has an identical morphology to that of the R-SLIP leg. In this case, the translational DOF of the robot is not used, and the leg-wheel motion is determined purely by the spring dynamics and the controlled leg orientation. This method is the sole method used in our previous work on generating tripod running by the RHex-style robot since that robot has only one rotational DOF on each leg and no translational DOF available. These two methods are described in separate subsections as follows.

A. Using the 2-DOF Mechanism

The leg-wheel module of the *TurboQuad* has 2-DOF motion moving according to the polar coordinate as shown in the upper left corner of Fig. 5. This mechanism allows the robot to be operated in 1-DOF wheeled motion, 2-DOF legged motion, and for leg-wheel transformation. Based on the strategy described in previous section, the work here is to actuate the 2-DOF leg to move like the R-SLIP leg with one torsion spring.

The quantitative mapping between the 2-DOF leg-wheel and the passive dynamic compression of the R-SLIP leg can be accomplished by using the geometrical relation as shown

in Fig. 5. The exact mapping is complex, but the differential representation of the mapping can be obtained. The rotational leg motion θ and translational motion ΔR of the leg-wheel can be linked to the rotational angle φ and spring compression angle ϕ of the R-SLIP model:

$$\theta = \varphi \quad (2)$$

$$\Delta R = R(\cos(\rho) - \cos(\rho - \phi) + (\sin(\rho) + \sin(\rho - \phi))\tan(\varphi)) \quad (3)$$

where ρ is the parameter determining position of the torsion spring. For *TurboQuad*, it is set to 65 degrees away from the mass according to past experience (i.e. $\rho = 65$).

One exemplary comparison of the original R-SLIP mass trajectory and the simulated trajectory generated by (2) and (3) is plotted in Fig. 5(b). The x and y components of the R-SLIP mass trajectory cannot be simultaneously matched using the geometrical mapping because of the constraint $\theta = \varphi$. In the figure, the trajectory in the y-direction is closely matched, but that in the x-direction has about a 0.06 m error at maximum.

As the spring effect is simulated by the motor motion, ideally, if the motor power is sufficient, the 2-DOF mechanism can simulate the compression effect of torsion springs with a wide range of stiffness. This is advantageous because it provides the possibility of actuating the legs with different passive compliance characteristics in real time.

B. Using a Torsion Spring

Using the R-SLIP model as a template suggests that the overall behavior of the robot's leg can be represented by the passive dynamics of the torsion spring. Thus, in addition to using the 2-DOF mechanism, the leg-wheel can be modified to match the R-SLIP leg as shown in Fig. 6. Due to empirical space and spring fabrication constraints, three springs are installed in parallel in the final design. In addition, this method also allows the changing of the stiffness in the follow-up use. When the robot is operating in this method, the translational DOF of the leg-wheel is fixed to its maximum

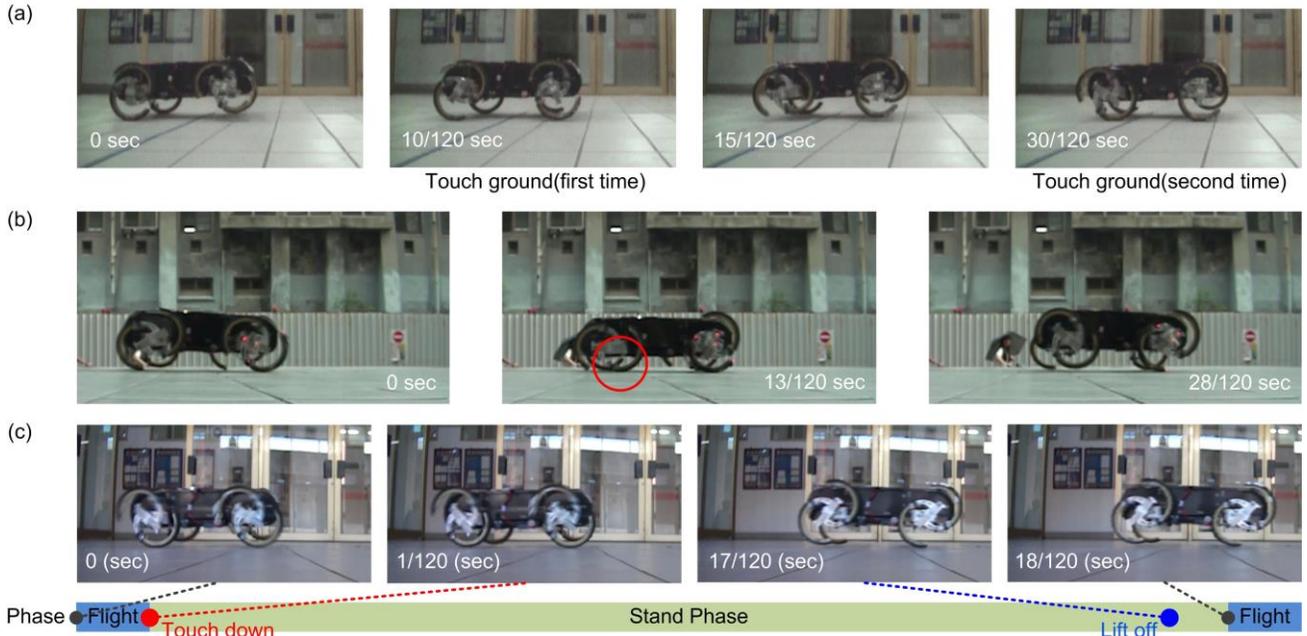


Fig. 7. The robot performs (a) trotting locomotion using the 2-DOF mechanism method, (b) trotting locomotion using the torsion spring method, and (c) pronking locomotion using the torsion spring method.

length, which has the shortest offset distance between the hip joint and top of the leg-wheel. This reduces the error caused by a configuration mismatch between the robot leg and the model leg. By using the passive torsion spring, the mechanism can react rapidly to the external force. In addition, by actively using only the rotational DOF, this method also reduces the loading of the DC motors.

IV. PERFORMANCE EVALUATION

The performance of the proposed model-based strategy was experimentally evaluated on the robot. The experiment was conducted on flat tiled ground and was recorded by a camcorder in $0.25 \times$ slow record mode.

Figure 7(a) plots snapshots of the robot with a trotting gait generated by the 2-DOF mechanism method. The figure clearly reveals that the robot contacts the ground twice in one stand phase of the R-SLIP model, where the R-SLIP model only exhibits a single contact. The first ground contact takes place when the leg touches the ground. Ideally, if the robot moves according to the passive dynamics of the model, the leg would retract to simulate the torsion spring compression of the R-SLIP model. However, if the robot's state does not perfectly match the ideal condition, any mismatch would cause the robot's leg to impact the ground. The rebound force due to the impact caused the leg to depart from the ground. As the force is not significant enough to let the robot's body fly, the robot's leg would make a second ground contact shortly after the first. As a result, the robot has double contacts in one stance phase, and this unwanted impact behavior deviates the robot's behavior from the designed R-SLIP motion. To remedy this, the robot should have either a sensory mechanism to detect the ground contact condition or a

compliance control strategy to generate soft contact. On the other hand, the robot with a pronking gait generated by the 2-DOF method is not executed owing to the constraint of the motor speed. The corresponding leg trajectory changes too fast for the current motor to track. This difficulty might be solvable by searching for different fixed points that have lower speed and less spring stiffness.

Figure 7(b) plots snapshots of the robot with a trotting gait generated by the torsion spring method. The figure reveals that the motion pattern of the robot is quite similar to that of the R-SLIP model. However, the compressions of the leg torsion springs in the stand phase are so large that the other legs in the aerial phase touch the ground, which causes a disturbance to the robot's trotting. This problem mainly results from the empirical mechanical design, and so can be solved by providing a larger ground clearance when the legs are in the aerial phase.

The trotting behaviors generated by both methods have different behaviors. The leg reaction speed is faster in the torsion spring method than in the 2-DOF method, which indicates that a motor with higher power may be desired. In contrast, the torsion spring method allows the robot to have a fast response yet with soft contact. Thus, we can consider this as the best method from a design approach, which relies more on mechanical structure than on control effort. On the other hand, the designed stiffness of the torsional spring cannot be easily changed, which limits the variation range of gait development because different fixed points generally have different operating leg stiffness. In this respect, the 2-DOF method is beneficial.

Figure 7(c) plots snapshots of the robot with a pronking gait generated by the torsion spring method. The figure shows that

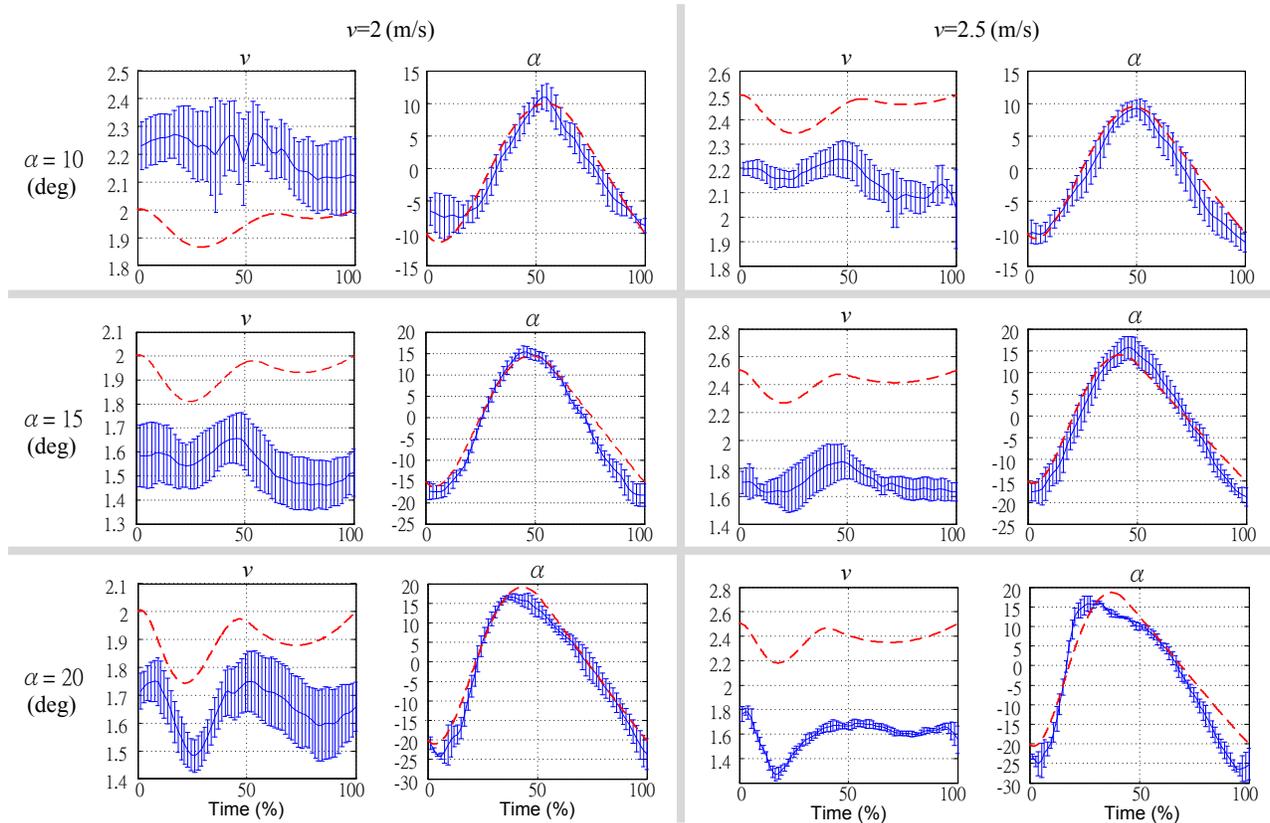


Fig. 8. Ground truth data while the robot performs pronking locomotion using the torsion spring method.

the robot performs the pronking behavior quite well, and the unwanted leg contact observed in Fig. 7(b) is not observed in this figure. This is mainly because the robot operating in pronking gait uses all four legs simultaneously. Thus, the resultant stiffness of the pronking robot is higher than that of the trotting robot. Although the robot running with two different gaits uses different fixed points, the fixed points are intentionally selected to be similar, which yields a similar energy exchange pattern. Since the potential energy of the pronking robot is stored in all four springs, the spring compression level is less than that of the trotting robot and is, therefore, free from the unwanted leg contact problem.

The white LED installed on top of the robot is programmed to indicate the nominal phase condition (i.e., stance or flight) of the robot. If the “on” and “off” of the LED matches the empirical robot’s ground contact condition, the robot runs at the natural dynamic of the R-SLIP model. Figure 7(c) and the supplementary video associated with this paper reveal that the timing is really close (error within 4%), so the robot is excited with the desired dynamics.

Figure 8 plots the state of the robot with a pronking gait generated by the torsion spring method. The amplitude and direction of the CoM velocity of the robot were measured using the ground truth measurement system. The robot runs with six different initial conditions, including two touchdown speeds ($v=2, 2.5$ m/s) and three touchdown angles ($\alpha=10, 15, 20$ degree). Five experimental runs are collected for each

setting, except for the case of $v=2.5$ m/s and $\alpha=20$ degrees. The red dashed curves represent the nominal trajectory of the R-SLIP model, and the blue curves in the middle and vertical bars indicate the mean and standard deviation of the data, respectively. Figure 8 indicates that the touchdown angles of the robot with all initial conditions are very close to those of the model. As for the touchdown velocities, those of the robot have a similar trend to those of the model, but the absolute magnitude has some discrepancy. This is mainly due to the slippage between the ground-contact legs and the ground as well as the limitation of motor power.

Table II shows the mean and the standard deviation (std) of the robot’s roll and pitch angles. Every set of experiments is repeated three times and at least five complete periods are recorded. The small angles confirm that the robot has a very small pitch and roll variation during the pronking motion. Thus, the body’s inertial effect can be neglected. In this case, the body dynamics can be simplified to be treated as a point mass. Therefore, using the 2-DOF reduced-order model to represent the sagittal-plane motion of the robot is adequate.

The experiment results confirm that the robot can excite dynamic legged motion, and that the motion pattern can be further designed or predicted by the R-SLIP model. The experimental results also reveal that the torsion spring method can be used to generate both a trotting and pronking gait. To prevent the unwanted leg collision with the ground, a torsion spring with a higher stiffness is desired. Note that when the

TABLE II STATISTICAL SUMMARY OF THE ROBOT'S PITCH AND ROLL IN EXPERIMENTS

Initial conditions		Roll angle (deg)	Pitch angle (deg)
v (m/s)	α (degree)	Mean(std)	Mean(std)
2	10	-1.58(2.87)	-1.69(1.64)
2	15	-0.33(1.77)	-0.39(1.72)
2	20	-0.07(0.55)	-1.49(1.69)
2.5	10	-0.81(1.77)	-1.37(1.38)
2.5	15	-0.23(0.50)	-0.50(2.18)
2.5	20	-0.13(0.82)	0.01(2.28)

desired stiffness of the robot with a trotting gait changes, that with a pronking gait changes as well. A design trade-off may be faced whereby the increased stiffness may not be favorable in generating the pronking gait since the stiffness may be overlarge.

V. CONCLUSION

We report on the model-based development of dynamic trotting and pronking gaits on a leg-wheel transformable robot. The stable and dynamic R-SLIP motion is utilized as the desired and nominal CoM trajectory of the robot. In order to match the robot leg motion to the model leg motion, the 2-DOF mechanism method and the torsion spring method are investigated. The experimental results reveal that the 2-DOF method would require a feedback mechanism or compliance control to compensate for the disturbance acting on the empirical robot. In addition, the pronking gait also requires a lower spring stiffness and speed. The experimental results further reveal that the passive compliant properties of the leg indeed smoothen the contact impact, so the leg can be operated without additional control effort. However, the stiffness in this method is set at the hardware level, so the gait change simultaneously causes the resultant leg stiffness change. This indicates that the fixed points for robot operation are not freely selectable, but are instead a function of existing gait and mechanical design. If suitable initial conditions and torsion spring stiffness for different gaits exist, the 2-DOF mechanism method might be a good and simple solution. The experimental results confirm that the robot can excite its pronking behavior quite stably and that the behavior of the robot has a high similarity to that of the R-SLIP model, except for the existence of certain ground slippage. Compared to the tuning or optimization method where each cost setting yields one result, the model-based methodology allows us to program the robot's dynamic behaviors with different state conditions or gaits in one design process.

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